Tailoring the emergence of many-body physics with trapped-ion crystals

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Outline

- Motivation
- Quantum Ising Ladders
- Synthetic gauge fields
- Magneto-structural phase transitions
1. Motivation

*Main Goal:* Use trapped ions as a platform to study quantum many-body physics

*Why* quantum many-body physics?

1. *Because* there are plenty of exotic phenomena that *emerge* out of the collective behaviour of these systems (e.g. Quantum Hall effect, High-Tc superconductivity, etc) some of which are still considered *open problems*.

*Why* trapped ions and not some solid-state material?

*Because* trapped ions (among other AMO setups) offer the unique possibility to tailor the microscopic Hamiltonian to study these effects in a controllable and clean setup, sometimes surpassing the possibilities of real materials.
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1. Motivation

Task-oriented quantum simulator → A quantum system capable of reproducing the physics of a particular Hamiltonian $H$

Task: Mimic the system Hamiltonian $H$

Resources: Coupling strengths of the system Hamiltonian $H_{\text{sys}}(g_1, g_2 \cdots)$

Protocol: Tuning of the coupling strengths

\[ H_{\text{sys}} \xrightarrow{\{g_i\} \rightarrow \{\tilde{g}_i\}} H_{\text{eff}} \approx H \]

Eq. Effective Hamiltonian

By tuning some experimental knobs, we can build a desired Hamiltonian, and simulate the physics of interesting models (e.g. condensed-matter, high-energy physics, etc.) in a controlled atomic table-top experiment.
3 steps to QS

1. Prepare initial states

More More More!!

2. Adiabatic evolution

\[ \{ g_2(t) \} \]

3. Measure final states

|gs⟩ |gs⟩ |gs⟩ |gs⟩
|gs⟩ |gs⟩ |gs⟩ |gs⟩ |gs⟩ |gs⟩
1. Motivation

Main Goal: Use trapped ions as a platform for the QS of many-body physics

There are several proposals and experimental realization of many-body QSs with trapped ions


Reviews on QS's
1. Motivation

I will cover three different topics to extend the trapped-ion QS toolbox

a. Quantum Ising ladders (Frustrated Magnetism)

b. Synthetic gauge fields (Quantum Hall physics)

c. Magneto-structural phase transitions (Spin-Peierls)
2. Coulomb crystals of trapped ions

Coulomb Crystals are self-assembled structures of trapped ions

$$H = \sum_{j=1}^{N} \sum_{\alpha=x,y,z} \left( \frac{1}{2m} p_{j\alpha}^2 + \frac{1}{2} m \omega^2 r_{j\alpha}^2 \right) + \frac{e^2}{2} \sum_{j \neq k} \frac{1}{|\mathbf{r}_j - \mathbf{r}_k|}$$

equilibrium positions


N. Kjaergaard et al., PRL 91, 095002 (2003).

By controlling the trapping frequencies in a Paul trap, we can create ladders of bond-sharing triangles with any desired number of legs.

\[ \kappa_x = \left( \frac{\omega_z}{\omega_x} \right)^2 \]

\[ n_1 = 1 \]

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\[ n_1 = 3 \]

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\[ \begin{align*}
  n_1 &= 1 \\
  n_1 &= 2 \\
  n_1 &= 4
\end{align*} \]
Harmonic approximation.- The small vibrations around the equilibrium positions are coupled by the Coulomb interaction → Collective phonon modes

\[ H_p = \sum_{n\lambda} \Omega_{n\lambda} a_{n\lambda}^\dagger a_{n\lambda} \]

Two-level approximation.- The atomic energy structure presents several levels, two of which can be addressed by lasers → Pseudo-spins = 1/2
Harmonic approximation.- The small vibrations around the equilibrium positions are coupled by the Coulomb interaction → Collective phonon modes

Two-level approximation.- The atomic energy structure presents several levels, two of which can be addressed by lasers → Pseudo-spins $s=1/2$

\[ H_p = \sum_{n\lambda} \Omega_{n\lambda} a_{n\lambda}^{\dagger} a_{n\lambda} \]

\[ H_s = \frac{1}{2} \delta \sum_i \sigma_i^z - \frac{1}{2} \Omega \sum_i \sigma_i^x \]
How can we exploit the different degrees of freedom

a. spins
b. phonons
c. structural

to perform many-body quantum simulations?
3. Quantum Ising ladders

1a. It is difficult to understand how the interplay of quantum fluctuations and frustration can stabilise new phases of matter (e.g. quantum spin liquids).
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\[ H_{\text{class}} = \sum_{\langle i,j \rangle} |J_{i,j}| \sigma_i^z \sigma_j^z \]

Exponential degeneracy of the groundstate manifold

How is this degeneracy lifted by quantum fluctuations? Does a new phase emerge out of this degenerate manifold?
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Exponential degeneracy of the groundstate manifold

$$H_{\text{quant}} = \sum_{\langle i,j \rangle} |J_{ij}| \sigma_i^z \sigma_j^z - \sum_i h \sigma_i^x$$

How is this degeneracy lifted by quantum fluctuations?

Does a new phase emerge out of this degenerate manifold?

(reminiscent of the FQHE)
3. Quantum Ising ladders

It is difficult to find materials that realise low-dimensional quantum Ising models.

"... a real-life constraint is that most spins are vectors and not scalar Ising spins as frequently used in models..."


Is it possible to design the range of Ising interactions in different lattices and control the quantum fluctuations?
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and even more difficult that those materials display a variable range of frustration

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Is it possible to design the range of Ising frustration in different lattices and control the quantum fluctuations?
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Spin-phonon couplings can be tailored by means of an atomic Lambda scheme.
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\( \omega_L \) determines the spin-phonon coupling

\[ \omega_L \approx \Omega_{n, \lambda} \ll \omega_0 \]

crossed-beam ac-Stark shift

\[ H_d = \frac{\Omega_L}{2} \sum_j \sigma_j^z e^{i k_L \cdot r_j^0} e^{i (k_L \cdot q_j^0 - \omega_L t)} + \text{H.c.} \]

spin phonon

first exp. realization in the context of quantum-state engineering \( e^S \) logic gates

3. Quantum Ising ladders

Quantum Ising models can be obtained by phonon-mediated interactions

\[ H_d = \frac{\Omega_L}{2} \sum_j \sigma_j^z e^{i \mathbf{k}_L \cdot \mathbf{r}_j^0} e^{i (\mathbf{k}_L \cdot \mathbf{q}_j^0 - \omega_L t)} + \text{H.c.} \]

\[ \omega_L \approx \Omega_{n\lambda} \ll \omega_0 \]

When the phonons are only virtually excited, they act as modifiers of an Ising coupling

\[ H_{\text{ph}} = \sum H_{\text{ph} x} + H_{\text{ph} y} \]

\[ k_{ij} = k_i - k_j \quad \text{decouples the coupled phonon polarization} \]

\[ \begin{cases} J_{ij} > 0 & k_i \perp e_\perp \\ J_{ij} < 0 & k_i \parallel e_\parallel \\ J_{ij} = 0 & \end{cases} \]
3. Quantum Ising ladders

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\[ H_{\text{eff}} = \sum_{i,j} J_{ij} \sigma_i^z \sigma_j^z - h \sum_i \sigma_i^x \]

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\[ H_{\text{eff}} = \sum_{i,j} J_{ij} \sigma_i^x \sigma_j^z - h \sum_i \sigma_i^x \]


\[ k_L = k_1 - k_2 \]

determines the coupled phonon “polarization”

\[ \left\{ \begin{array}{c}
J_{ij} > 0 \quad k_L \perp \mathbf{e}_z \\
J_{ij} < 0 \quad k_L \parallel \mathbf{e}_z
\end{array} \right. \]

“Directional selection rule”
Our idea is to substitute this "directional selection rule" for an "energetic selection rule".
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\[ H_{\text{eff}} = \sum_{i \neq j} J_{ij}^{\text{eff}} \sigma_i^z \sigma_j^z - \hbar \sum_i \sigma_i^x \]

\[ J_{ij}^{\text{eff}} \propto \cos \left( k_L \cdot \left( r_i^0 - r_j^0 \right) \right) / \left| r_i^0 - r_j^0 \right|^3 \]
We obtain a quantum simulator for frustrated quantum Ising Ladders

\[ H_{\text{leg}} = \sum_{\gamma} \sum_{i_s \neq j_s} J_{i_s; j_s}^{\gamma} \sigma_{i_s}^{\gamma \cdot} (\gamma) \sigma_{j_s}^{\gamma \cdot} (\gamma) - h \sum_{\gamma} \sum_{i_s} \sigma_{i_s}^{\gamma x} (\gamma) \]

\[ H_{\text{Ising}} = \sum_{\gamma} \sum_{i_s, j_s} J_{i_s; j_s}^{\gamma \cdot} \sigma_{i_s}^{\gamma \cdot} (\gamma) \sigma_{j_s}^{\gamma \cdot} (\gamma) \]

In the diagram, we see a lattice with interactions labeled as \( J \) and \( h \). The Hamiltonian \( H_{\text{leg}} \) represents the leg Hamiltonian for the Ising model, which can be tuned to represent frustrated Ising models.
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\[ H_{\text{rung}} = \sum_{\gamma \neq \mu} \sum_{i_s \neq j_s} \tilde{J}_{i_s, j_s}^{\gamma, \mu} \sigma_{i_s}^{z}(\gamma) \sigma_{j_s}^{z}(\mu). \]
We obtain a quantum simulator for frustrated quantum Ising Ladders.

\[ H_{\text{leg}} = \sum_{\gamma} \sum_{i_s \neq j_s} J^\gamma_{i_s, j_s} \sigma^z_{i_s}(\gamma) \sigma^z_{j_s}(\gamma) - \hbar \sum_{\gamma} \sum_{i_s} \sigma^x_{i_s}(\gamma) \]

\[ H_{\text{rung}} = \sum_{\gamma \neq \mu} \sum_{i_s \neq j_s} J^\gamma,\mu_{i_s, j_s} \sigma^z_{i_s}(\gamma) \sigma^z_{j_s}(\mu). \]

\[ J^\gamma,\mu_{i_s, j_s} / J^\gamma_{i_s, j_s} \in (-\infty, \infty) \quad \rightarrow \text{we can tune the range of frustration} \]
We obtain a quantum simulator for frustrated quantum Ising Ladders.

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It is even more difficult to find materials with a variable range of Ising frustration.
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1b It is even more difficult to find materials with a variable range of Ising frustration possible to synthesize.
Overview of our results


We have checked the validity of our scheme by comparing the effective Ising ladder Hamiltonian with exact numerics taking into account sources of error (dephasing noise, thermal fluctuations, photon scattering).
Overview of our results


We have studied numerically the phase diagram of the Quantum Ising zigzag ladder (J1-J2 quantum Ising model), and found evidence supporting a new ordered phase due to the long-ranged frustration mechanism.
Overview of our results


We have discussed a method to change the geometry of the ladders to that of corner-sharing triangles (sawtooth chain, stripes of the Kagome lattice), which could lead to a quantum simulator of quantum spin liquids and quantum dimer models.

Overview of our results


We have optimized the trap geometry so that the required anisotropy of the trap frequencies can be varied over a wide range (experimental proof of principle). This will allow the electronic control of the range of frustration.
4. Synthetic gauge fields

It is very difficult to understand how the interplay of strong magnetic fields with interactions and disorder can stabilize exotic phases of matter (e.g. topological order)

\[ B = \nabla \times A \]

\[ H = -t \sum_{\langle r, r' \rangle} c_r^\dagger e^{-i \int A \, dl} c_{r'} + \text{h.c.} \]
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\[ \Phi = \frac{1}{\Phi_0} \int B \mathrm{d}S \quad \# \text{flux quanta per unit cell} \]

Macroscopic degeneracy of the groundstate manifold (Lowest Landau level)
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How is this degeneracy lifted by interactions and disorder?
Does a new phase emerge out of this degenerate manifold?

(reminiscent of the FQIM)
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2b. It is very difficult to produce controllable magnetic fields leading to \( \Phi \approx 1 \) in typical materials (tiny unit cells).
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This underlies the reason why the full phase diagram of the *Hofstadter Topological Insulator* has not been observed yet.
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This underlies the reason why the full phase diagram of the Hofstadter Topological Insulator has not been observed yet.

and even more difficult that those materials display a variable strength of interactions and disorder

Is it possible to realize these models with the phonon excitations of trapped-ion crystals?
Tight-binding model (TBM) for the vibrational excitations.- The harmonic approximation leading to the normal vibrational modes can be expressed as a TBM for the local vibrational excitations

\[ H = \sum_{\alpha, n} \Omega_{\alpha n} a^\dagger_{\alpha n} a_{\alpha n} \rightarrow \text{normal-mode phonons} \]

Local vibrational excitations, "local phonons"
Tight-binding model (TBM) for the vibrational excitations. - The harmonic approximation leading to the normal vibrational modes can be expressed as a TBM for the local vibrational excitations

\[ H = \sum_{\alpha, n} \Omega_{n, \alpha} a_{n, \alpha}^\dagger a_{n, \alpha} \rightarrow \text{normal-mode phonons} \]


\[ H = H_0 + H_c = \sum_{i, \alpha} \omega'_{\alpha, i} b_{\alpha, i}^\dagger b_{\alpha, i} + \sum_{\alpha} \sum_{i > j} (J_{c; ij}^\alpha b_{\alpha, i}^\dagger b_{\alpha, j} + \text{H.c.}) \]

local vibrational excitations, “local phonons”
Tight-binding model (TBM) for the vibrational excitations. - The harmonic approximation leading to the normal vibrational modes can be expressed as a TBM for the local vibrational excitations.

Can we control the tunneling to mimic a Peierls substitution?

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local vibrational excitations, "local phonons"
Photon-assisted tunneling.- The amplitude of the tunneling strengths can be controlled by introducing a **gradient of the trap frequencies** together with a **periodic modulation**

\[ H_0(t) = (\omega_\alpha + \eta_d \omega_d \cos(\omega_d t)) b_{\alpha,j}^\dagger b_{\alpha,j} + (\omega_\alpha + \Delta \omega_\alpha) b_{\alpha,i}^\dagger b_{\alpha,i}. \]

**periodic driving assists the tunneling**  
\[ \omega_d = \Delta \omega \]

**gradient inhibits the tunneling**  
\[ |J| \ll \Delta \omega \]
Photon-assisted tunneling. - The amplitude of the tunneling strengths can be controlled by introducing a gradient of the trap frequencies together with a periodic modulation.

\[ H_{\text{eff}} = J_{d;ij}^\alpha b_{\alpha,i}^\dagger b_{\alpha,j} + \text{H.c.}, \]

\[ J_{d;ij}^\alpha \approx J_{c;ij}^\alpha J_0(\eta_d) \]

dressed tunneling

Bessel function

We can tune \( \eta_d \) to zero of the Bessel function to cancel the tunneling.

Coherent destruction of tunneling
dynamical localization
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Coherent destruction of tunneling dynamical localization


Is it possible to exploit this effect to mimic a synthetic gauge field?
Synthetic gauge fields. - If the periodic modulation has a site-dependent phase, the dressed tunnelings acquire a Peierls phase

\[ H_0(t) = \sum_{\alpha, i} \left( \omega_\alpha + \Delta \omega_\alpha i_1 + \eta_d \omega_d \cos(\omega dt + \phi_i) \right) b_{\alpha, i} \dagger b_{\alpha, i}. \]
Synthetic gauge fields.- If the periodic modulation has a site-dependent phase, the dressed tunnelings acquire a Peierls phase

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\[ H_{\text{eff}} = \sum_{\alpha} \sum_{i>j} \tilde{f}_\alpha^{ij} e^{ie^* f_j^i dr \cdot A} b^\dagger_{\alpha,i} b_{\alpha,j} + \text{H.c.,} \]
Synthetic gauge fields. - If the periodic modulation has a site-dependent phase, the dressed tunnelings acquire a Peierls phase

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$$H_{\text{eff}} = \sum_{\alpha} \sum_{i>j} \tilde{f}_d^\alpha_{i;j} e^{ie^{*} \int_j^{i} dr \cdot A_s} b_{\alpha,i}^\dagger b_{\alpha,j} + \text{H.c.,}$$

$$W_{\phi}^{(1)} \propto e^{ie^{*} \int d r \cdot A_s} = e^{ie^{*} \int \Box B_s \cdot d S}.$$ 

$$\phi_{\phi} = \phi_2 \quad \# \text{flux quanta per unit cell} \approx 1$$
Synthetic gauge fields.- If the periodic modulation has a site-dependent phase, the dressed tunnelings acquire a Peierls phase

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\[ H_{\text{eff}} = \sum_{\alpha} \sum_{i>j} \tilde{f}_{d,ij} \alpha e^{i \phi_{\alpha}} \int d^r A_s b_{\alpha,i}^\dagger b_{\alpha,j} + \text{H.c.}, \]

\[ W_{\phi}^{(1)} \propto e^{i \phi_{\alpha}} \int d^r A_s = e^{i \phi_{\alpha}} \int \Box B_s \cdot dS. \]

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How can we implement such a periodic driving?
Periodic drivings via dipole forces.- We proposed to use the atomic Lambda scheme in a different regime.
Periodic drivings via dipole forces.- We proposed to use the atomic Lambda scheme in a different regime

\[ \omega_L \ll \Omega_{\eta,\lambda} \ll \omega_0 \]

crossed-beam periodic modulation

\[ H_L \approx -\Omega_L \sum_{\alpha,i} \eta_\alpha^2 \cos(\omega_L t - \mathbf{k}_L \cdot \mathbf{r}_i^0) b_{\alpha,i}^\dagger b_{\alpha,i}. \]

site-dependent phase

2b It is very difficult to produce controllable magnetic fields leading to in typical materials (tiny unit cells)
Periodic drivings via dipole forces.- We proposed to use the atomic Lambda scheme in a different regime.

\[ \omega_L \text{ determines the spin-phonon coupling} \]

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possible to synthesize

It is very difficult to produce controllable magnetic fields leading to in microtrap arrays of trapped ions

2b
Overview of our results


We have checked the validity of our scheme by comparing the effective gauged Hamiltonian with the exact dynamics for an Aharonov-Bohm interference also taking into account finite temperatures.
Overview of our results


We have studied the possibility of tailoring gauged rhombic ladders, leading to the phenomena of edge states typical of topological insulators.

Overview of our results


We have extended the toolbox of this QS to incorporate the effects of:

- **on-site interactions** → non-linearities due to a standing wave (FQHE)
- **diagonal disorder** → randomness introduced by the spin degrees of freedom through a detuned red sideband
- **off-diagonal disorder** → randomness in the dressed tunnelings introduced by a spin-dependent periodic modulation
- **non-Abelian gauges** → opposite gradients for different vibrational directions (flavours) lead to SU(2) gauge fields (QSHE)
- **Gauge flux lattices** → arbitrary patterns of synthetic fluxes through a detuned red sideband
5. Magneto-structural phase transitions

It is very difficult to understand completely the physics of spin-phonon coupled quantum magnets

\[ H = \sum \omega_n a_n^\dagger a_n + \sum_{\langle i,j \rangle} |J_{ij}| \left( 1 + \sum_n \xi_n (a_n^\dagger + a_n) \right) \sigma_i^x \sigma_j^z - h \sum_i \sigma_i^x \]
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It is very difficult to understand completely the physics of spin-phonon coupled quantum magnets

\[ H = \sum \omega_n a_n^\dagger a_n + \sum_{\langle i,j \rangle} |J_{ij}| \left( 1 + \sum_n \xi_n (a_n^\dagger + a_n) \right) \sigma_i^x \sigma_j^x - \hbar \sum_i \sigma_i^x \]

The condensation of a particular phonon mode (i.e. structural phase transition) may lead to an order-disorder magnetic phase transition (and viceversa)

spin-version of Peierls instability 1D metals

R. E. Peierls, Quantum Theory of Solids (1955),
5. Magneto-structural phase transitions

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spin-version of Peierls instability 1D metals

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What happens with non-adiabatic effects (no separation of scales for spin and phonons)?
5. Magneto-structural phase transitions

It is very difficult to find materials that realize spin-phonon coupled quantum Ising models.

Heisenberg Peierls samples

- TTF-CuS$_4$C$_4$(CF$_3$)$_4$
- TTF-AuS$_4$C$_4$(CF$_3$)$_4$
- CuGeO$_3$
- TiOCl


and even more difficult that these materials display a variable range of adiabaticity.
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Heisenberg Peierls samples

\[ \frac{|J_{ij}|}{\omega_n} \in [0, \infty) \]

and even more difficult that those materials display a variable range of adiabaticity

|J_{ij}|

faster phonon excitations

faster spin excitations

5. Magneto-structural phase transitions

It is very difficult to find materials that realize spin-phonon coupled quantum Ising models.

Heisenberg Peierls samples

\[
\begin{align*}
TTF-CuS_4C_4(CF_3)_4 \\
TTF-AuS_4C_4(CF_3)_4 \\
CuGeO_3 \\
TiOCl
\end{align*}
\]


\[|J_{ij}|/\omega_n \in [0, \infty)\]

and even more difficult that those materials display a variable range of adiabaticity

Is it possible to design the range of phonon/spin adiabaticity?

faster phonon excitations

\[\rightarrow\]

faster spin excitations
5. Magneto-structural phase transitions

So far, we have focused only on the stable regions in between the structural phase transitions
5. Magneto-structural phase transitions

Now, we would like to study what happens close to criticality.
**Inhomogeneous $\phi^4$-model.** Close to the first structural phase transition $\kappa_{c,2}$, there is a soft phonon mode (zigzag distortion) that condenses

$$q_{ix} = (-1)^i \delta q_{zz}^i, \quad \langle \delta q_{zz}^i \rangle \neq 0$$

**Inhomogeneous \( \phi^4 \)-model.** - Close to the first structural phase transition \( \kappa_{c,2} \), there is a soft phonon mode (zigzag distortion) that condenses

\[
q_{ix} = (-1)^i \delta q_{iz}^{zz} \quad \langle \delta q_{iz}^{zz} \rangle \neq 0
\]


\[H_x = \sum_i \left( \frac{m l_z^2}{2} (\partial_x \delta q_{iz}^{zz})^2 + \frac{r_i^x}{2} (\delta q_{iz}^{zz})^2 + \frac{u_i^x}{4} (\delta q_{iz}^{zz})^4 \right) + \sum_{i \neq j} \frac{K_{ij}^x}{2} (\partial_x \delta q_{iz}^{zz})^2\]

\[r_i^x > 0 \quad \kappa_{c,2} \quad r_i^x < 0\]
Peierls spin-phonon coupling. - Let's revisit our "energetic selection rules"
Peierls spin-phonon coupling. - Let’s revisit our “energetic selection rules”

Only the transverse phonons mediate the interaction. Thus, the planar modes can be integrated out with the exception of the zigzag mode since it condenses at the SPT.

Can we exploit this to synthesize a spin-phonon coupled Ising magnet?
Peierls spin-phonon coupling. To exploit the zigzag mode, we use the atomic Lambda scheme in yet a different regime.
Peierls spin-phonon coupling. - To exploit the zigzag mode, we use the atomic Lambda scheme in a different regime.

\[ \omega_L \approx \omega_0 \pm \Omega_{\text{nl}} \]

crossed-beam Raman transition

\[ H_d = \frac{\Omega_{\text{nl}}}{2} \sum_j \sigma_j^+ e^{i k_L \cdot r_j} e^{i (k_L \cdot q_j - \omega_L t)} + \text{H.c.} \]

The combination of the two subbands, after electromagnetically eliminating the transverse phonons leads to an effective long-range model in a different basis:

The expression for the interaction Hamiltonian is:

\[ \Pi_{\text{nl}} = \sum_{l \eta} \frac{\Omega_{\text{nl}}}{2} \sigma^+_l \sigma^\eta_l = \hbar \sum_{l \eta} \sigma^+_l \sigma^\eta_l \]
Peierls spin-phonon coupling. - To exploit the zigzag mode, we use the atomic Lambda scheme in a different regime

\[ \omega_L \approx \omega_0 \pm \Omega_{\mu \lambda} \]

crossed-beam Raman transition

\[ H_d = \frac{\Omega_L}{2} \sum_j \sigma_j^+ e^{i k_L \cdot r_j^0} e^{i (k_L \cdot q_j^0 - \omega_L t)} + \text{H.c.} \]

The combination of the two sidebands with opposite detunings leads to an effective QIM

\[ H_{\text{QIM}} = \sum_{i \neq j} J_{i,j}^{\text{eff}} \sigma_i^x \sigma_j^x - \hbar \sum_i \sigma_i^z \]

Our idea is to exploit an extra degree of freedom = the relative orientation between the effective wavevectors of each of the Raman beams

\[ H_{\text{eff}} = \sum_{i \neq j} \left( J_{ij}^{xx} \sigma_i^x \sigma_j^x + J_{ij}^{yy} \sigma_i^y \sigma_j^y + J_{ij}^{xy} \sigma_i^x \sigma_j^y + J_{ij}^{yx} \sigma_i^y \sigma_j^x \right), \]

\[ k^r_L = -k^b_L \]

The effect of the condensing phonon mode translates into a spin-phonon coupled magnet

\[ H_{\text{QIM}} = \sum_{i \neq j} J_{ij}^{\text{eff}} \sigma_i^x \sigma_j^x - h \sum_i \sigma_i^z \]

\[ H_{\text{dimer}} = \sum_{i \neq j} J_{ij}^{\text{dim}} \sigma_i^x \sigma_j^y + J_{ij}^{\text{dim}} \sigma_i^y \sigma_j^x \]

It is very difficult to find materials that realize spin-phonon coupled QIM.
Our idea is to exploit an extra degree of freedom = the relative orientation between the effective wavevectors of each of the Raman beams

\[ H_{\text{eff}} = \sum_{i \neq j} \left( \frac{J_{ij}^{xx}}{\sigma_i^x} \sigma_i^x \sigma_j^x + \frac{J_{ij}^{yy}}{\sigma_i^y} \sigma_i^y \sigma_j^y + \frac{J_{ij}^{xy}}{\sigma_i^x} \sigma_i^y \sigma_j^y + \frac{J_{ij}^{yx}}{\sigma_i^y} \sigma_i^x \sigma_j^y \right), \]

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possible to synthesize

It is very difficult to find materials that realize spin-phonon coupled QIM
The effect of the condensing phonon mode translates into a spin-phonon coupled magnet.

\[ H_{\text{dimer}} = \sum_{i \neq j} J_{ij}^{\text{dim}} \sigma_i^x \sigma_j^x + J_{ij}^{\text{dim}} \sigma_i^y \sigma_j^y \]

\[ J_{ij}^{\text{dim}} \propto (-1)^j \delta q_j^{zz} \]

The structural phase transition induces dimerizes spin couplings.

The diagrams illustrate the transition from one phase to another, with \( J_{ij}^{\text{dim}} \) being the spin coupling and \( \delta q_j^{zz} \) being the phonon displacement.
The effect of the condensing phonon mode translates into a spin-phonon coupled magnet

\[ H_{\text{dimer}} = \sum_{i \neq j} J_{ij}^{\text{dim}} \sigma_i^x \sigma_j^x + J_{ij}^{\text{dim}} \sigma_i^y \sigma_j^x \]

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The structural phase transition induces dimerizes spin couplings

Can we correlate the SPT with the magnetic order-disorder QPT*?

Can we find a spin-Peierls transition only driven by the transverse field (i.e. quantum fluctuations) \( g = \frac{h}{J} \)?

cooperative
Jahn-Teller effect

The effect of the condensing phonon mode translates into a spin-phonon coupled magnet

\[ H_{\text{dimer}} = \sum_{i \neq j} J_{ij}^{\text{dim}} \sigma_i^x \sigma_j^y + J_{ij}^{\text{dim}} \sigma_i^y \sigma_j^x \]

\[ J_{ij}^{\text{dim}} \propto (-1)^j \delta q_j^{zz} \]

The structural phase transition has an associated Paramagnet-Antiferromagnet QPT

**Spin-Peierls Quantum Phase Transition**

**Linear Paramagnet**

\[ g > \tilde{g}_c \]

**Zigzag Antiferromagnet**

\[ g < \tilde{g}_c \]

only driven by quantum fluctuations
Overview of our results


We have analyzed the spin-Peierls transition close to the SPT (slow zigzag mode). The spin system can be solved analytically, compared to numerical DMRG solution, and then incorporated self-consistently in the $\phi^4$ model.
Overview of our results


- We have identified regimes where the spin-Peierls QPT can be driven solely by quantum fluctuations.
- We have identified regimes where the adiabatic separation of phonon/spin dynamics will not be valid (open problem)
Thanks for your attention!!
Micromotion Contributions

Micromotion = fast motion of the ions synchronous with the trap rf frequency

\[ r_{i\alpha} = r_{i\alpha}^0 \left( 1 + \frac{1}{2} q_{\alpha} \cos(\Omega_{\text{rf}}t) \right) + \Delta r_{i\alpha}(t) \left( 1 + \frac{1}{2} q_{\alpha} \cos(\Omega_{\text{rf}}t) \right), \]

excess micromotion  \quad secular motion  \quad q_{\alpha} \sim 0.1 - 0.2

\[ |r_{i\alpha}^0| \gg |\Delta r_{i\alpha}(t)| \]

Excess micromotion can be very large when eq. positions lie off the trap axis. However, its error contribution to the spin QS can be controlled

a) Micromotion heating

\[ \Delta H(t) = \frac{\omega_z}{2} \sum_{i,j} \kappa_{ij}^{1/2} \tilde{V}_{yy}(t) a_{i,y}^{\dagger} a_{j,y}^{\dagger} e^{2i\omega_y t} + \text{H.c.}, \]

Coulomb coupling considering excess micromotion
Micromotion Contributions

Micromotion = fast motion of the ions synchronous with the trap rf frequency

\[ r_{i\alpha} = r_{i\alpha}^0 \left( 1 + \frac{1}{2} q_\alpha \cos(\Omega_{rf} t) \right) + \Delta r_{i\alpha}(t) \left( 1 + \frac{1}{2} q_\alpha \cos(\Omega_{rf} t) \right), \]

excess micromotion

secular motion

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Excess micromotion can be very large when eq. positions lie off the trap axis. However, its error contribution to the spin QS can be controlled

a) Micromotion heating

\[ \kappa_{\gamma y}^{1/2} |\mathbf{v}_{ij}^{yy}| \ll \frac{|2\omega_y \pm \Omega_{rf}|}{\omega_z} \approx \frac{\Omega_{rf}}{\omega_z}. \]

avoids creation of phonons (i.e. heating)
Micromotion Contributions

Micromotion = fast motion of the ions synchronous with the trap rf frequency

\[ r_{i\alpha} = r_{i\alpha}^0 \left( 1 + \frac{1}{2} q_\alpha \cos(\Omega_{rf} t) \right) + \Delta r_{i\alpha}(t) \left( 1 + \frac{1}{2} q_\alpha \cos(\Omega_{rf} t) \right), \]

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b) Micromotion contribution to spin-dependent forces

\[ V(t) = \sum_{l,s} \sum_i \frac{1}{2} \Omega_{l,s} |r_i \rangle \langle s_i| e^{i k \cdot r_i} e^{i \xi_i(t)} e^{i \delta_i(t)} + H.c., \]

micromotion sidebands

\[ \xi_{li} = q_x k_l \cdot \frac{r_{i\alpha}^0}{2} \]

\[ |\xi_{li}| \ll 1. \]

laser conf
Micromotion Contributions

Micromotion = fast motion of the ions synchronous with the trap rf frequency

\[ r_{i\alpha} = r_{i\alpha}^0 \left( 1 + \frac{1}{2} q_\alpha \cos(\omega_{rf} t) \right) + \Delta r_{i\alpha}(t) \left( 1 + \frac{1}{2} q_\alpha \cos(\omega_{rf} t) \right), \]

excess micromotion \quad secular motion

\[ q_\alpha \sim 0.1 - 0.2 \]
\[ |r_{i\alpha}^0| \gg |\Delta r_{i\alpha}(t)| \]

Excess micromotion can be very large when eq. positions lie off the trap axis. However, its error contribution to the spin QS can be controlled

b) Micromotion contribution to spin-dependent forces

\[ V(t) \propto \sum_{l,s,i} \sum_m \frac{1}{2} \Omega_{l,s} J_m(\xi_{li}) |r_i\rangle \langle s_i| e^{i k_{li} \cdot r_i} e^{i(\delta_{l,s} - m\omega_{rf})t} + \text{H.c.} \]

\[ \Omega_{rf}/2\pi \approx 0.1 \text{GHz} \ll \Delta/2\pi \approx 10 \text{GHz}, \quad m \gg 1 \]
\[ J_m(\xi_{li}) \propto (\xi_{li})^m \ll 1 \]
Micromotion Contributions

b) Micromotion contribution to spin-dependent forces

\[ \epsilon_m = \max_t \left\{ |\langle \tilde{\sigma}^x(t) \rangle_{\text{mic}} - \langle \tilde{\sigma}^x(t) \rangle_{\text{eff}}|, \, t \in \left[ 0, \frac{6\pi}{\Omega_L} \right] \right\} , \]
Micromotion Contributions

c) Micromotion contribution to unwanted transitions

\[ \Delta V(t) = \sum_{i,i,a} \frac{\Omega_{i,a}}{2} |r_i\rangle \langle a_i| e^{i k \cdot r_i} e^{i \xi_i t} e^{i \delta_{i,a} t} + \text{H.c.,} \]

micromotion sidebands might take us out of the spin manifold \( a_i \neq \{ \uparrow_i, \downarrow_i \} \) via 2-photon processes

\[ \Omega_{1,a} \Omega_{2,s}^* J_m(\xi_{1i}) J_{m'}(\xi_{2i})^* e^{-i(\delta_{a,\uparrow} - \omega_L - (m-m')\Omega_{rf}) t} \]

we can minimize it by controlling Zeeman shifts

\[ |\Omega_{1,a} \Omega_{2,s}^*| \ll |\delta_{a,s} - \omega_L \pm \Omega_{rf}|, \quad |\xi_{li}| \ll 1. \]
Quantum Dimer models

Quantum dimer models introduced in the context of high-Tc superconductivity (undoped cuprates) & Heisenberg models also arise for quantum Ising models

dimer = frustrated bond

For the J1-J2 QIM \( H = -|J_1| \left( \sum_i \sigma_i^z \sigma_{i+1}^z - f_2 \sum_i \sigma_i^z \sigma_{i+2}^z + g \sum_i \sigma_i^x \right) \),

\( |\text{dAF}\rangle \in \text{span} \{ \left| \begin{array}{c} \uparrow \downarrow \cdots \uparrow \downarrow \end{array} \right\rangle, \left| \begin{array}{c} \uparrow \uparrow \cdots \uparrow \uparrow \end{array} \right\rangle, \left| \begin{array}{c} \downarrow \uparrow \cdots \downarrow \uparrow \end{array} \right\rangle, \left| \begin{array}{c} \downarrow \downarrow \cdots \downarrow \downarrow \end{array} \right\rangle \} \). dimer coverings
Quantum Dimer models

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\(|dAF\rangle \in \text{span} \left\{ |\uparrow \uparrow \cdots \uparrow \rangle, |\downarrow \downarrow \cdots \downarrow \rangle, |dimer coverings\rangle \right\}.

When each spin is connected to the same number of satisfied & broken bonds

dimers may start to resonate due to the transverse field of the QIM (similar RVB)
Q-dimer model (e.g. triangular lattice)
Quantum Spin Liquids

Quantum fluctuations not always lead to the uninteresting paramagnet. Sometimes they stabilize more exotic phases that do not break any symmetry of the Hamiltonian spin liquid phases

\[ H_{\Delta\Delta} = J_1 \sum_i \sigma_i^z \sigma_{i+1}^z + J_2 \sum_i \sigma_{2i-1}^z \sigma_{2i+1}^z - h \sum_i \sigma_i^x. \]

minimal conjectures spin liquid phase \( \rightarrow \) sawtooth QIM

\( h > 0 \rightarrow \) no symmetry breaking

Quantum Spin Liquids

Quantum fluctuations not always lead to the uninteresting paramagnet. Sometimes they stabilize more exotic phases that do not break any symmetry of the Hamiltonian spin liquid phases

higher-dimensional spin liquid phase $\rightarrow$ Kagome QLM

Kagome Stripe
**J1-J2 quantum Ising model**

The short-range J1-J2 QIM

\[
H = -|J_1| \left( \sum_i \sigma_i^z \sigma_{i+1}^z - f_2 \sum_i \sigma_i^z \sigma_{i+2}^z + g \sum_i \sigma_i^x \right),
\]

frustration range

\[f_2 = J_2/|J_1|\]

Simple phases:

**Ferromagnetic** \(|F\rangle \in \text{span}\{\left|\uparrow \uparrow \cdots \uparrow \uparrow\right\rangle, \left|\downarrow \downarrow \cdots \downarrow \downarrow\right\rangle\}, \quad g = 0, f_2 < \frac{1}{2}\)

**Dimerized AFM** \(|d\text{AF}\rangle \in \text{span}\{\left|\uparrow \downarrow \cdots \uparrow \downarrow\right\rangle, \left|\downarrow \uparrow \cdots \downarrow \uparrow\right\rangle\}, \quad g = 0, f_2 > \frac{1}{2}\)

**Paramagnet** \(|P\rangle = |\rightarrow \rightarrow \cdots \rightarrow \rightarrow\rangle\), \quad g \gg 1 \gg f_2\)
The short-range J1-J2 QIM

\[ H = -|J_1| \left( \sum_i \sigma_i^z \sigma_{i+1}^z - f_2 \sum_i \sigma_i^z \sigma_{i+2}^z + g \sum_i \sigma_i^x \right), \]

frustration range

\[ f_2 = J_2 / |J_1| \]

critical incommensurate phase

\[ \langle \sigma_i^z \sigma_j^z \rangle_{FP} \sim m_0^2 \cos(q_{FP}(i-j))|i-j|^{-\eta} \]
**J1-J2 quantum Ising model**

The dipolar-range J1-J2 QIM

\[ H_{\Delta\Delta} = -|J_1| \left( \sum_i \sum_{\delta \in \text{odd}} f_\delta \sigma_i^x \sigma_{i+\delta}^x - \sum_i \sum_{\delta \in \text{even}} f_\delta \sigma_i^z \sigma_{i+\delta}^z + \sum_i g \sigma_i^x \right), \quad f_\delta = J_\delta / |J_1| \]

The dipolar-range couplings lead to competing frustration sources

---

**Diagram:**

**a** dAF short range

**b** dAF long range

---
**J1-J2 quantum Ising model**

The dipolar-range J1-J2 QIM

\[
H_{\Delta \delta} = -|J_1| \left( \sum_i \sum_{\delta \in \text{odd}} f_{\delta} \sigma_i^z \sigma_{i+\delta}^z - \sum_i \sum_{\delta \in \text{even}} f_{\delta} \sigma_i^z \sigma_{i+\delta}^z + \sum_i g \sigma_i^x \right), \quad f_{\delta} = J_{\delta} / |J_1|
\]

The dipolar-range couplings lead to competing frustration sources.
Spin-Peierls quantum phase transition

The spin-phonon system consists of

\[ H_x = \sum_i \left( \frac{ml_z^2}{2} (i \delta q^{zz}_i)^2 + \frac{r_i^x}{2} \left( \delta q^{zz}_i \right)^2 + \frac{w_i^x}{4} \left( \delta q^{zz}_i \right)^4 \right) + \sum_{i \neq j} \frac{K_{ij}^x}{2} (\partial_j \delta q^{zz}_i)^2 \]

\[ H_{QIM} = \sum_{i \neq j} J_{ij}^{eff} \sigma_i^x \sigma_j^x - \hbar \sum_i \sigma_i^z \]

\[ H_{\text{dim}} = \sum_{i \neq j} (-1)^j \delta q^{zz}_{ij} (\sigma_i^x \sigma_j^y + \sigma_i^y \sigma_j^x) \]

phonon condensation = dimerized spin couplings

The dimerization lowers the magnetic energy of the AF groundstate, supporting the energy cost of the lattice structural change
Spin-Peierls quantum phase transition

We have solved this magneto-structural phase transition in the limit of slow phonons (sort of adiabatic approximation + mean-field theory)

\[
H_x = \sum_i \left( \frac{\mu_{i}^2}{2} (\partial_i \delta q_{ix})^2 + \frac{r_{i}^x}{2} (\delta q_{ix})^2 + \frac{u_{i}^x}{4} (\delta q_{ix})^4 \right) + \sum_{i \neq j} \frac{K_{ij}^x}{2} (\partial_i \delta q_{ij}^z)^2
\]

\[
H_{\text{QIM}} = \sum_{i \neq j} J_{ij}^{\text{eff}} \sigma_i^x \sigma_j^x - \hbar \sum_i \sigma_i^z
\]

\[
H_{\text{dim}} = \sum_{i \neq j} (-1)^j \langle \delta q_{j}^{zz} \rangle (\sigma_i^x \sigma_j^y + \sigma_i^y \sigma_j^x)
\]

The short-range spin model can be solved via Jordan-Wigner + Bogoliubov transformation

\[
g_c \rightarrow \tilde{g}_c(\xi) = \sqrt{1 + 4\xi^2}.
\]

instability towards AFM \( \xi \propto \langle \delta q_{ij}^{zz} \rangle \)
Spin-Peierls quantum phase transition

We have solved this magneto-structural phase transition in the limit of slow phonons (sort of adiabatic approximation + mean-field theory)

\[
H_x = \sum_i \left( \frac{m_i^2}{2} (\partial_i \delta q_{ix})^2 + \frac{r_i^x}{2} (\delta q_{ix})^2 + \frac{u_i^x}{4} (\delta q_{ix})^4 \right) + \sum_{i\neq j} \frac{K_{ij}^x}{2} (\partial_i \delta q_{ij}^z)^2
\]

\[
H_{QIM} = \sum_{i \neq j} J_{ij}^{\text{eff}} \sigma_i^x \sigma_j^x - \hbar \sum_i \sigma_i^z
\]

\[
H_{\text{dim}} = \sum_{i \neq j} (-1)^j \langle \delta q_{j}^{zz} \rangle (\sigma_i^x \sigma_j^y + \sigma_i^y \sigma_j^x)
\]

The short-range spin model can be solved via Jordan-Wigner + Bogoliubov transformation

\[
E_g(\xi) \approx E_g(0) - \frac{2JN}{\pi} \varphi(g) \xi^2 < E_g(0), \quad \xi \propto \langle \delta q_{j}^{zz} \rangle
\]

self-consistency
Spin-Peierls quantum phase transition

We have solved this magneto-structural phase transition in the limit of slow phonons (sort of adiabatic approximation + mean-field theory)

\[ H_x = \sum_i \left( \frac{m_i^2}{2} \left( \partial_i \delta q_{ix} \right)^2 + \frac{r_i^x}{2} \langle \delta q_{ix} \rangle^2 + \frac{u_i^x}{4} \langle \delta q_{ix} \rangle^4 \right) + \sum_{i \neq j} \frac{K_{ij}^x}{2} \left( \partial_i \delta q_{ijx} \right)^2 \]

\[ r_i^x \rightarrow \tilde{r}_i^x \]

\[ \kappa_{c,i} \rightarrow \tilde{\kappa}_{c,i} = \left( \frac{\zeta_i(3)}{2} + \frac{2J}{m \omega^2 l_z^2} \frac{\theta^2 \varphi(g)}{\pi} \right)^{-1} \]

instability towards zigzag ladder

The short-range spin model can be solved via Jordan-Wigner + Bogoliubov transformation

\[ E_g(\xi) \approx E_g(0) - \frac{2JN}{\pi} \varphi(\xi) \xi^2 < E_g(0), \quad \xi \propto \langle \delta q_{ix} \rangle \]

self-consistency
Spin-Peierls quantum phase transition

We have solved this magneto-structural phase transition in the limit of slow phonons (sort of adiabatic approximation + mean-field theory)

Linear Paramagnet  
\[ g > \tilde{g}_c \]  
Zigzag Antiferromagnet  
\[ g < \tilde{g}_c \]

only driven by quantum fluctuations
Photon-assisted tunneling

We consider the gradient + periodic driving with a site-dependent phase

$$H_0(t) = \sum_{\sigma, i} (\omega_\sigma + \Delta \omega \delta_{1i} + \eta_d \omega_d \cos(\omega_d t + \phi_i)) a_{\sigma,i}^\dagger a_{\sigma,i}.$$ 

In the Interaction picture with respect to this term

$$a_{\sigma,j}(t) = e^{-i(\omega_\sigma + \Delta \omega j_1) t} e^{-i \eta_d \sin(\omega_d t + \phi_j)} a_{\sigma,j}.$$ 

$$a_{\sigma,i}^\dagger(t) = e^{+i(\omega_\sigma + \Delta \omega i_1) t} e^{+i \eta_d \sin(\omega_d t + \phi_i)} a_{\sigma,i}^\dagger.$$ 

Their product in the tunneling terms $a_{\sigma,i}^\dagger a_{\sigma,j}$ does not cancel
Photon-assisted tunneling

By using the Jacobi-Augé expansion in terms of Bessel’s functions

\[ e^{i \eta_d \sin(\omega_d t + \phi_i)} = \sum_{s \in \mathbb{Z}} J_s(\eta_d) e^{i s(\omega_d t + \phi_i)}, \]

it can be shown that the terms that dress the tunneling fulfill a resonance condition

\[ r \omega_d = \Delta \omega_\sigma \]

ion absorbs \( r \) photons to accomplish the tunneling

which leads to

\[ H_{\text{eff}} = \sum_{\sigma} \sum_{i > j} J^\sigma_{d;ij} a^\dagger_{\sigma,i} a_{\sigma,j} + \text{H.c.,} \]

\[ J^\sigma_{d;ij} = J^\sigma_{t;ij} F_f(i,j)(\eta_d, \eta_d, \Delta \phi_{ij}) e^{-i \frac{f(i,j)}{2}(\phi_i + \phi_j)}, \]

amplitude modulation

gauge field
Photon-assisted tunneling

\[ J_{d;ij}^\sigma = J_{t;ij}^\sigma F_f(i,j) (\eta_d, \eta_d, \Delta \phi_{ij}) e^{-i \frac{f(i,j)}{2} (\phi_1 + \phi_j)}, \]

amplitude modulation
Photon-assisted tunneling

\[ J_{d;ij}^\sigma = J_{t;ij}^\sigma \mathcal{F}_f(i,j)(\eta_d, \eta_d, \Delta \phi_{ij})e^{-i\frac{f(i,j)}{2}(\phi_1 + \phi_j)}, \]

gauge field
\[ \phi_i = \phi_1 i_1 + \phi_2 i_2 \]

Wilson loop
\[ W_\sigma^{(1)} = J_{d;1,i+e_2}^\sigma J_{d;i+e_2,i+e_1+e_2}^\sigma J_{d;i+e_1+e_2,i+e_1}^\sigma J_{d;i+e_1,i}^\sigma \]

arbitrarily strong synthetic gauge fluxes
\[ W_\sigma^{(1)} = |W_\sigma^{(1)}|e^{i\phi_\sigma} \]
\[ \phi_\sigma = r\phi_2 \]